Chapter 3

Computation with positive and negative integers

What you will learn

3A Working with negative integers
3B Adding or subtracting a positive integer
3C Adding or subtracting a negative integer
3D Multiplying or dividing by an integer
3E Order of operations with positive and negative integers
3F The Cartesian plane
The coldest place on Earth

The coldest place on Earth is Antarctica, where maximum daily temperatures below 0°C Celsius are commonplace. The coldest temperature ever recorded on Earth was about –89°C in 1983 at the Russian Vostok Station in Antarctica. Temperatures on the coast of Antarctica can reach 15°C in summer but average temperatures are generally below 0°C. A layer of ice surrounds and covers almost all of the continent. The surrounding ice sits both above and below sea level, making it very difficult for ships to reach their destinations. Like the temperature scale, heights above and below sea level can be recorded using both positive and negative numbers.
Chapter 3 Computation with positive and negative integers

1. Insert the symbols < (is less than) or > (is greater than) to make each statement true.
   a. 5 < 7  
   b. 0 < 10  
   c. 9 > 11  
   d. 3 < 0

2. Read the temperature on these thermometers.
   a. °C
      -20  -15  -10  -5  0  5  10  15  20
   b. °C
      -10  -6  -2  0  2  4  8  10
   c. °C
      -10  -2  0  2  4  6  8  10

3. Evaluate these products.
   a. 2 × 15  
   b. 11 × 7  
   c. 3 × 13  
   d. 28 × 4

4. Evaluate these quotients.
   a. 35 ÷ 7  
   b. 121 ÷ 11  
   c. 84 ÷ 12  
   d. 340 ÷ 20

5. Use order of operations to evaluate the following.
   a. 2 + 5 × 4  
   b. 10 ÷ 2 − 3  
   c. (11 + 15) × 2  
   d. 24 ÷ (8 − 2)  
   e. (6 − 3) × (1 + 9)  
   f. 8 × (4 − 2) + 10 ÷ 5

6. Decide if the answers to these expressions are positive (i.e. greater than zero) or negative (i.e. less than zero).
   a. 5 − 4  
   b. 4 − 5  
   c. 10 × 2 − 21  
   d. 30 − 5 × 4

7. Write down the coordinates (x, y) of A, B and C for this Cartesian plane.

8. Plot these points on the given Cartesian plane.
   a. A (2, 3)  
   b. B (4, 1)  
   c. C (5, 4)  
   d. D (0, 2)  
   e. E (3, 0)
The numbers 1, 2, 3, … are considered to be positive because they are greater than zero (0). Negative numbers extend the number system to include numbers less than zero. All the whole numbers less than zero, zero itself and the whole numbers greater than zero are called integers.

The use of negative numbers dates back to 100 BC when the Chinese used black rods for positive numbers and red rods for negative numbers in their rod number system. These coloured rods were used for commercial and tax calculations. Later, a great Indian mathematician named Brahmagupta (598–670) set out the rules for the use of negative numbers, using the word 

fortune

for positive and 

debt

for negative. Negative numbers were used to represent loss in a financial situation.

An English mathematician named John Wallis (1616–1703) invented the number line and the idea that numbers have a direction. This helped define our number system as an infinite set of numbers extending in both the positive and negative directions. Today negative numbers are used in all sorts of mathematical calculations and are considered to be an essential element of our number system.

Let’s start: Simple applications of negative numbers

• Try to name as many situations as possible in which negative numbers are used.
• Give examples of the numbers in each case.

Key ideas

- **Negative** numbers are numbers less than zero.
- **Integers** are whole numbers that can be negative, zero or positive.
  
  … −4, −3, −2, −1, 0, 1, 2, 3, 4, …
- The number −4 is read as ‘negative 4’.
- The number 4 is sometimes written as +4 and is sometimes read as ‘positive 4’.
- Every number has direction and magnitude.
- A number line shows:
  
  - positive numbers to the right of zero
  - negative numbers to the left of zero.
- A thermometer shows:
  
  - positive temperatures above zero
  - negative temperatures below zero.
- Each number other than zero has an opposite.
  
  3 and −3 are examples of opposite numbers. They are equal in magnitude but opposite in sign.
Chapter 3 Computation with positive and negative integers

Example 1 Drawing a number line

Draw a number line, showing all integers from −4 to 2.

**SOLUTION**

-4, −3, −2, −1, 0, 1, 2

**EXPLANATION**

Use equally spaced markings and put −4 on the left and 2 on the right.

Example 2 Less than or greater than

Insert the symbol < (is less than) or > (is greater than) into these statements to make them true.

**a** −2 □ 3

**b** −1 □ −6

**SOLUTION**

**a** −2 < 3

−2 is to the left of 3 on a number line.

**b** −1 > −6

−1 is to the right of −6 on a number line.

Exercise 3A

1. What are the missing numbers on these number lines?

   **a**
   
   −3 □ −1 □ 0 □ 1 □ 3

   **b**
   
   −2 □ −1 □ 1 □ 3

   **c**
   
   −10 −9 −8 □ −6 □

   **d**
   
   □ −4 □ −2 □ −1 □

2. −5 is the opposite number of 5, and 5 is the opposite number of −5. Write down the opposite to these numbers.

   **a** 2  
   **b** 6  
   **c** −3  
   **d** −7  
   **e** −15  
   **f** 21  
   **g** 132  
   **h** −1071

3. Fill in the blanks using the words greater and less.

   **a** 5 is ___________ than 0
   **b** −3 is ___________ than 0
   **c** 0 is ___________ than −6
   **d** 0 is ___________ than 1
4 Draw a number line for each description, showing all the given integers.
   a from –2 to 2  b from –5 to 1  c from –10 to –6  d from –32 to –25

5 List all the integers that fit the given description.
   a from –2 up to 4  b from –7 up to 0
   c greater than –3 and less than 2  d greater than –5 and less than 1
   e less than 4 and greater than –4  f less than –3 and greater than –10

6 Insert the symbol < (is less than) or > (is greater than) into these statements to make them true.
   a 7 9  b 3 2  c 0 –2  d –4 0
   e –1 –5  f –7 –6  g –11 –2  h –9 –13
   i –3 3  j 3 –3  k –130 1  l –2 –147

7 Give the temperature for these thermometers.
   a °C  b °C  c °C  d °C
   ![Thermometers]

8 Arrange these numbers in ascending order.
   a –3, –6, 0, 2, –10, 4, –1  b –304, 126, –142, –2, 1, 71, 0

9 Write the next three numbers in these simple patterns.
   a 3, 2, 1, ___, ___, ___  b –8, –6, –4, ___, ___, ___
   c 10, 5, 0, ___, ___, ___  d –38, –40, –42, ___, ___, ___
   e –91, –87, –83, ___, ___, ___  f 199, 99, –1, ___, ___, ___

10 These lists of numbers show deposits (positive numbers) and withdrawals (negative numbers) for a month of bank transactions. Find the balance at the end of the month.
   a Starting balance $200
      –$10
      –$130
      $25
      –$100
      $20
      Final balance _____
   b Starting balance $0
      $50
      –$60
      –$100
      $200
      –$100
      Final balance _____
11 If the height above sea level for a plane is a positive number, then the height for a submarine could be written as a negative number. What is the height relative to sea level for a submarine at these depths?

- a 50 metres
- b 212.5 metres
- c 0 metres

12 The difference between two numbers could be thought of as the distance between the numbers on a number line. For example, the difference between $-2$ and $1$ is 3.

\[ \begin{array}{c}
-3 & -2 & -1 & 0 & 1 & 2 & 3 \\
3 & & & & & & \\
\end{array} \]

Find the difference between these pairs of numbers.

- a $-1$ and $1$
- b $-2$ and $2$
- c $-3$ and $1$
- d $-4$ and $3$
- e $-3$ and $0$
- f $-4$ and $-1$
- g $-10$ and $-4$
- h $-30$ and $14$

**Enrichment: The final position**

13 For these sets of numbers, a positive number means to move right and a negative number means to move left. Start at zero each time and find the final position.

\[ \begin{array}{c}
-3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\text{negative} & & & & & & \\
\text{positive} & & & & & & \\
\end{array} \]

- a $-1$, $4$, $-5$
- b $3$, $-5$, $-1$, $4$
- c $-5$, $-1$, $3$, $1$, $-2$, $-1$, $4$
- d $-10$, $20$, $-7$, $-14$, $8$, $-4$
- e $-250$, $300$, $-49$, $-7$, $36$, $-81$
- f $-7001$, $6214$, $-132$, $1493$, $-217$
Adding or subtracting a positive integer

Adding and subtracting a positive integer can give both positive and negative answers. For example, when the temperature inside a newly installed fridge is 20°C but, after being switched on, the temperature then falls by 25°C, the result is −5°C; i.e. 20 − 25 = −5. If a temperature of −10°C rises by 5°C, the result is −5°C; i.e. −10 + 5 = −5.

Let’s start: Positive and negative possibilities

Decide if it is possible to find an example of the following. If so, give a specific example.

- A positive number added to a positive number gives a positive number.
- A positive number added to a positive number gives a negative number.
- A positive number added to a negative number gives a positive number.
- A positive number added to a negative number gives a negative number.
- A positive number subtracted from a positive number gives a positive number.
- A positive number subtracted from a positive number gives a negative number.
- A positive number subtracted from a negative number gives a positive number.
- A positive number subtracted from a negative number gives a negative number.

■ If a positive number is added to a number, you move right on a number line.

2 + 3 = 5  Start at 2 and move right by 3.

−5 + 2 = −3  Start at −5 and move right by 2.

■ If a positive number is subtracted from a number, you move left on a number line.

2 − 3 = −1  Start at 2 and move left by 3.

−4 − 2 = −6  Start at −4 and move left by 2.
Example 3 Adding and subtracting positive integers

Calculate the answer to these additions and subtractions.

a \(-2 + 3\)  
b \(-8 + 1\)  
c \(5 - 7\)  
d \(-3 - 3\)

**SOLUTION**

**EXPLANATION**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(-2 + 3 = 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>(-8 + 1 = -7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>(5 - 7 = -2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>(-3 - 3 = -6)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Exercise 3B**

1. In which direction (i.e. right or left) on a number line do you move for the following calculations?
   a. 2 is added to \(-5\)
   b. 6 is added to \(-4\)
   c. 4 is subtracted from 2
   d. 3 is subtracted from \(-4\)

2. Match up the problems a to d with the number lines A to D.
   a. \(5 - 6 = -1\)
   b. \(-2 + 4 = 2\)
   c. \(-1 - 3 = -4\)
   d. \(-6 + 3 = -3\)
3 Calculate the answer to these additions. Check your answers using a calculator.

\[
\begin{align*}
a &\quad -1 + 2 \\
b &\quad -1 + 4 \\
c &\quad -3 + 5 \\
d &\quad -10 + 11 \\
e &\quad -4 + 3 \\
f &\quad -5 + 2 \\
g &\quad -11 + 9 \\
h &\quad -20 + 18 \\
i &\quad -4 + 0 \\
j &\quad -8 + 0 \\
k &\quad -30 + 29 \\
l &\quad -39 + 41 \\
m &\quad -130 + 132 \\
n &\quad -181 + 172 \\
o &\quad -57 + 63 \\
p &\quad -99 + 68
\end{align*}
\]

4 Calculate the answer to these subtractions. Check your answers using a calculator.

\[
\begin{align*}
a &\quad 4 - 6 \\
b &\quad 7 - 8 \\
c &\quad 3 - 11 \\
d &\quad 1 - 20 \\
e &\quad -3 - 1 \\
f &\quad -5 - 5 \\
g &\quad -2 - 13 \\
h &\quad -7 - 0 \\
i &\quad -37 - 4 \\
j &\quad 39 - 51 \\
k &\quad 62 - 84 \\
l &\quad -21 - 26 \\
m &\quad -100 - 200 \\
n &\quad 100 - 200 \\
o &\quad 328 - 421 \\
p &\quad -496 - 138
\end{align*}
\]

5 Find the missing number.

\[
\begin{align*}
a &\quad \square + 2 = 7 \\
b &\quad -2 + \square = 7 \\
c &\quad -2 + \square = 3 \\
d &\quad -4 + \square = -2 \\
e &\quad 5 - \square = 0 \\
f &\quad 3 - \square = -4 \\
g &\quad -9 - \square = -12 \\
h &\quad -20 - \square = -30 \\
i &\quad -6 + \square = -1 \\
j &\quad -8 - \square = -24 \\
k &\quad \square + 1 = -3 \\
l &\quad \square + 7 = 2 \\
m &\quad \square - 4 = -10 \\
n &\quad \square - 7 = -20 \\
o &\quad \square + 6 = -24 \\
p &\quad \square - 100 = -213
\end{align*}
\]

6 Evaluate the following. Remember to work from left to right.

\[
\begin{align*}
a &\quad 3 - 4 + 6 \\
b &\quad 2 - 7 - 4 \\
c &\quad -1 - 4 + 6 \\
d &\quad -5 - 7 - 1 \\
e &\quad -3 + 2 - 7 + 9 \\
f &\quad -6 + 1 - 20 + 3 \\
g &\quad 0 - 9 + 7 - 30 \\
h &\quad -15 - 20 + 32 - 1
\end{align*}
\]

7 Determine how much debt remains in these financial situations.

\[
\begin{align*}
a &\quad \text{owes} \$300 \text{ and pays back} \$155 \\
b &\quad \text{owes} \$20 \text{ and borrows another} \$35 \\
c &\quad \text{owes} \$21\,500 \text{ and pays back} \$16\,250
\end{align*}
\]

8 a The reading on a thermometer measuring temperature rises 18°C from −15°C. What is the final temperature?
b The reading on a thermometer measuring temperature falls 7°C from 4°C. What is the final temperature?
c The reading on a thermometer measuring temperature falls 32°C from −14°C. What is the final temperature?

9 For an experiment, a chemical solution starts at a temperature of 25°C, falls to −3°C, rises to 15°C and then falls again to −8°C. What is the total change in temperature? Add all the changes together for each rise and fall.

10 An ocean sensor is raised and lowered to different depths in the sea. Note that −100 metres means 100 metres below sea level.

a If the sensor is initially at −100 metres and then raised to −41 metres, how far does the sensor rise?
b If the sensor is initially at −37 metres and then lowered to −93 metres, how far is the sensor lowered?
11 Give an example that suits the description.
   a A positive number subtract a positive number equals a negative number.
   b A negative number subtract a positive number equals a negative number.
   c A negative number add a positive number equals a positive number.
   d A negative number add a positive number equals a negative number.

12 a $a$ is a positive integer, $b$ is a positive integer and $a > b$. For each of the following, decide if the result will be positive, negative or zero.
   i $a + b$
   ii $a - b$
   iii $b - a$
   iv $a - a$

   b $a$ is a negative integer and $b$ is a positive integer. Decide if each of the following is always true.
   i $a + b$ is positive
   ii $a - b$ is negative

Enrichment: + or – combinations

13 Insert + or – signs into these statements to make them true.
   a $3 \quad 4 \quad 5 = 4$
   b $1 \quad 7 \quad 9 \quad 4 = -5$
   c $-4 \quad 2 \quad 1 \quad 3 \quad 4 = 0$
   d $-20 \quad 10 \quad 7 \quad 36 \quad 1 \quad 18 = -4$
   e $-a \quad b \quad a \quad b = 0$
   f $-a \quad a \quad 3a \quad b \quad b = a - 2b$

<table>
<thead>
<tr>
<th>Month</th>
<th>Mean Temperature °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Daily Minimum</td>
</tr>
<tr>
<td>Jan</td>
<td>-12.9</td>
</tr>
<tr>
<td>Feb</td>
<td>-11.6</td>
</tr>
<tr>
<td>Mar</td>
<td>-7.5</td>
</tr>
<tr>
<td>Apr</td>
<td>-1.8</td>
</tr>
<tr>
<td>May</td>
<td>3.8</td>
</tr>
<tr>
<td>Jun</td>
<td>8.4</td>
</tr>
<tr>
<td>Jul</td>
<td>10.7</td>
</tr>
<tr>
<td>Aug</td>
<td>9.7</td>
</tr>
<tr>
<td>Sep</td>
<td>5.2</td>
</tr>
<tr>
<td>Oct</td>
<td>-1.8</td>
</tr>
<tr>
<td>Nov</td>
<td>-9.4</td>
</tr>
<tr>
<td>Dec</td>
<td>-12.3</td>
</tr>
</tbody>
</table>

Positive and negative numbers are used in everyday life.
Adding or subtracting a negative integer

By observing patterns in number calculations, we can see the effect of adding and subtracting negative integers.

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 + 3 = 5</td>
<td>2 - 3 = -1</td>
</tr>
<tr>
<td>2 + 2 = 4</td>
<td>2 - 2 = 0</td>
</tr>
<tr>
<td>2 + 1 = 3</td>
<td>2 - 1 = 1</td>
</tr>
<tr>
<td>2 + 0 = 2</td>
<td>2 - 0 = 2</td>
</tr>
<tr>
<td>2 + (-1) = 1</td>
<td>2 - (-1) = 3</td>
</tr>
<tr>
<td>2 + (-2) = 0</td>
<td>2 - (-2) = 4</td>
</tr>
<tr>
<td>2 + (-3) = -1</td>
<td>2 - (-3) = 5</td>
</tr>
</tbody>
</table>

So adding −3 is equivalent to subtracting 3, and subtracting −3 is equivalent to adding 3.

Let’s start: Dealing with debt

Let −$10 represent $10 of debt. Can you write a statement (e.g. 5 + (−10) = −5) to represent the following financial situations?

- $10 of debt is added to a balance of $5.
- $10 of debt is added to a balance of −$5.
- $10 of debt is removed from a balance of −$15.

- Adding a negative number is equivalent to subtracting its opposite.
  \[ a + (-b) = a - b \]

- Subtracting a negative number is equivalent to adding its opposite.
  \[ a - (-b) = a + b \]

- On a number line, the effect of adding or subtracting a negative number is to reverse the direction of the operation.
Example 4 Adding and subtracting negative integers

Calculate the answer to these additions and subtractions.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>7 + (-2)</td>
<td>b</td>
<td>-2 + (-3)</td>
</tr>
</tbody>
</table>

**Solution**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>7 + (-2) = 7 - 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 5</td>
<td></td>
</tr>
</tbody>
</table>

Adding -2 is equivalent to subtracting 2.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>b</td>
<td>-2 + (-3) = -2 - 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= -5</td>
<td></td>
</tr>
</tbody>
</table>

Adding -3 is equivalent to subtracting 3.

<p>| | | |</p>
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<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>c</td>
<td>1 - (-3) = 1 + 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 4</td>
<td></td>
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</tbody>
</table>

Subtracting -3 is equivalent to adding 3.

<p>| | | |</p>
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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>d</td>
<td>-6 - (-2) = -6 + 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= -4</td>
<td></td>
</tr>
</tbody>
</table>

Subtracting -2 is equivalent to adding 2.

Exercise 3C

1 Write down the missing numbers in these sentences. The first one has been done for you.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>2 + 5 means that 5 is added to 2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>-3 + 6 means that [ ] is added to [ ].</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>1 + (-3) means that [ ] is added to [ ].</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>-7 + (-11) means that [ ] is added to [ ].</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e</td>
<td>5 - 3 means that [ ] is subtracted from [ ].</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f</td>
<td>-2 - 6 means that [ ] is subtracted from [ ].</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g</td>
<td>7 - (-3) means that [ ] is subtracted from [ ].</td>
<td></td>
</tr>
<tr>
<td></td>
<td>h</td>
<td>-7 - (-11) means that [ ] is subtracted from [ ].</td>
<td></td>
</tr>
</tbody>
</table>

2 Complete these sentences.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>Adding -4 is equivalent to subtracting [ ].</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>Adding -6 is equivalent to [ ] 6.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>Adding 5 is equivalent to subtracting [ ].</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d</td>
<td>Adding -11 is equivalent to [ ] 11.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>e</td>
<td>Subtracting -2 is equivalent to adding [ ].</td>
<td></td>
</tr>
<tr>
<td></td>
<td>f</td>
<td>Subtracting -7 is equivalent to [ ] 7.</td>
<td></td>
</tr>
</tbody>
</table>

3 State whether each of the following is true or false.

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2 + (-3) = 5</td>
<td>b</td>
<td>10 + (-1) = 9</td>
<td>c</td>
<td>-5 + (-3) = -8</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>e</td>
<td>5 - (-1) = 4</td>
<td>f</td>
<td>3 - (-9) = 12</td>
<td>g</td>
<td>2 - (-3) = 1</td>
</tr>
</tbody>
</table>
Example 4a,b

4 Calculate the answer to each of these additions. Check your answer using a calculator.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3 + (−2)</td>
<td>b</td>
<td>8 + (−3)</td>
<td>c</td>
</tr>
<tr>
<td>e</td>
<td>1 + (−4)</td>
<td>f</td>
<td>6 + (−11)</td>
<td>g</td>
</tr>
<tr>
<td>i</td>
<td>−2 + (−1)</td>
<td>j</td>
<td>−7 + (−15)</td>
<td>k</td>
</tr>
<tr>
<td>m</td>
<td>−7 + (−3)</td>
<td>n</td>
<td>−20 + (−9)</td>
<td>o</td>
</tr>
</tbody>
</table>

Example 4c,d

5 Calculate the answer to each of these subtractions. Check your answer using a calculator.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2 − (−3)</td>
<td>b</td>
<td>5 − (−6)</td>
<td>c</td>
</tr>
<tr>
<td>e</td>
<td>−5 − (−1)</td>
<td>f</td>
<td>−7 − (−4)</td>
<td>g</td>
</tr>
<tr>
<td>i</td>
<td>−4 − (−6)</td>
<td>j</td>
<td>−9 − (−10)</td>
<td>k</td>
</tr>
<tr>
<td>m</td>
<td>5 − (−23)</td>
<td>n</td>
<td>28 − (−6)</td>
<td>o</td>
</tr>
</tbody>
</table>

6 Find the missing number.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2 + □ = −1</td>
<td>b</td>
<td>3 + □ = −7</td>
<td>c</td>
</tr>
<tr>
<td>e</td>
<td>□ + (−10) = −11</td>
<td>f</td>
<td>□ + (−4) = 0</td>
<td>g</td>
</tr>
<tr>
<td>i</td>
<td>−1 − □ = 3</td>
<td>j</td>
<td>□ − (−3) = 7</td>
<td>k</td>
</tr>
<tr>
<td>m</td>
<td>5 − □ = 11</td>
<td>n</td>
<td>□ − (−2) = −3</td>
<td>o</td>
</tr>
</tbody>
</table>

7 Calculate the answer, working from left to right.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3 + (−2) + (−1)</td>
<td>b</td>
<td>2 + (−1) + (−6)</td>
<td>c</td>
</tr>
<tr>
<td>d</td>
<td>10 − (−6) + (−4)</td>
<td>e</td>
<td>−7 − (−1) + (−3)</td>
<td>f</td>
</tr>
<tr>
<td>g</td>
<td>−9 − (−19) + (−16)</td>
<td>h</td>
<td>−15 − (−20) + (−96)</td>
<td>i</td>
</tr>
<tr>
<td>j</td>
<td>−2 − (−3) − (−5)</td>
<td>k</td>
<td>−18 − (−16) − (−19)</td>
<td>l</td>
</tr>
</tbody>
</table>

8 A diver is at a height of −90 metres from the surface of the sea. During a diving exercise, the diver rises 50 metres, falls 138 metres and then rises once again by 35 metres. What is the diver’s final height from sea level?
9 A small business has a bank balance of \(-\$50\,000\). An amount of \$20\,000 of extra debt is added to the balance and, later, \$35\,000 is paid back. What is the final balance?

10 \$100 of debt is added to an existing balance of \$50 of debt. Later, \$120 of debt is removed from the balance. What is the final balance?

11 Here is a profit graph showing the profit for each month of the first half of the year for a bakery shop.

   a What is the profit for:
      i February?
      ii April?

   b What is the overall profit for the 6 months?

12 Complete these magic squares, using addition. The sum of each row, column and diagonal should be the same.

   a
   \[
   \begin{array}{ccc}
   \text{1} & \text{2} & \\
   \text{5} & & \\
   \text{4} & & \\
   \end{array}
   \]

   b
   \[
   \begin{array}{ccc}
   & & -6 \\
   -3 & -17 & \\
   & -7 & \\
   \end{array}
   \]
13 Write these sentences as mathematical statements, e.g. \(2 + (-3)\).
   a The sum of 3 and 4.
   b The sum of \(-2\) and \(-9\).
   c The difference between 5 and \(-2\).
   d The difference between \(-2\) and 1.
   e The sum of \(a\) and the opposite of \(b\).
   f The difference between \(a\) and the opposite of \(b\).

14 Simplify these numbers. Hint: In part a, \(-(4)\) is the same as \(0 - (-4)\).
   a \(-(-4)\)
   b \(-(-(-1))\)
   c \(-(-(-(-3))))\)

15 a If \(a\) is a positive number and \(b\) is a negative number, decide if each of the following statements is always true or false.
    i \(a + b\) is negative
    ii \(a - b\) is positive

   b If \(a\) is a negative number and \(b\) is a negative number, decide if each of the following statements is always true or false.
    i \(a + b\) is negative
    ii \(a - b\) is positive

   c If \(a\) and \(b\) are both negative numbers and \(b < a\), is \(a - b\) always positive? Give reasons.

**Enrichment: Have some fun!**

16 Write down the value of these expressions.
   a \(1 + 2 + 3 + 4 + 5 + \ldots + 99 + 100\)
   b \(1 - 2 + 3 - 4 + 5 - \ldots + 99 - 100\)
   c \(1 + 2 - 3 + 4 + 5 + 6 - 7 - 8 + 9 + 10 - 11 - 12 \ldots - 99 - 100\)

17 Use a spreadsheet to check your answers to Question 16.
Multiplying or dividing by an integer

The rules for multiplication and division of integers can be developed by considering repeated addition. For example: 4 groups of \(-3\) is \(-3 + (-3) + (-3) + (-3) = -12\).

So, \(4 \times (-3) = -12\).

Also, \(-3 \times 4 = -12\) since \(a \times b = b \times a\).

We also know that if \(5 \times 7 = 35\), then \(35 \div 7 = 5\).

Similarly, if \(4 \times (-3) = -12\) then \(-12 \div (-3) = 4\). This is saying there are 4 groups of \(-3\) in \(-12\), which we know from the repeated addition above.

Also, \(-12 \div 4 = -3\).

These examples give rise to the rules governing the multiplication and division of negative numbers.

Let's start: Patterns in tables

Complete this table of values for multiplication by noticing the patterns. What does the table of values tell you about the rules for multiplying negative integers?

<table>
<thead>
<tr>
<th>(\times)</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key ideas

- The product or quotient of two numbers of the same sign (i.e. positive or negative) is a positive number.
  
  So \(a \times b = ab\) and \(-a \times (-b) = ab\)
  
  e.g. \(3 \times 4 = 12\) or \(-3 \times (-4) = 12\)
  
  and \(a + b = \frac{a}{b}\) and \(-a + (-b) = \frac{a}{b}\)
  
  e.g. \(12 + 4 = 3\) or \(-12 + (-4) = 3\)

- The product or quotient of two numbers of the opposite sign (i.e. positive and negative) is a negative number.
  
  So \(-a \times b = -ab\) and \(a \times (-b) = -ab\)
  
  e.g. \(-3 \times 4 = -12\) or \(3 \times (-4) = -12\)
  
  and \(-a + b = \frac{-a}{b}\) and \(a + (-b) = \frac{-a}{b}\)
  
  e.g. \(-12 + 3 = -4\) or \(12 + (-3) = -4\)
**Example 5 Multiplying and dividing integers**

Calculate these products and quotients.

a $5 \times (-6)$  
b $-3 \times (-7)$  
c $-36 \div (-4)$  
d $-18 \div 9$

**SOLUTION**

a $5 \times (-6) = -30$  
  The two numbers are of opposite sign, so the answer is negative.

b $-3 \times (-7) = 21$  
  The two numbers are of the same sign, so the answer is positive.

c $-36 \div (-4) = 9$  
  Both numbers are negative, so the answer is positive.

d $-18 \div 9 = -2$  
  The two numbers are of opposite sign, so the answer is negative.

**Example 6 Working with mixed operations**

Work from left to right to find the answer to $-7 \times 4 \div (-2)$.

**SOLUTION**

$-7 \times 4 \div (-2) = -28 \div (-2)$

$= 14$  
  First, calculate $-7 \times 4$.  
  Then calculate $-28 \div (-2)$.

**Exercise 3D**

1 Complete these product tables.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\times$</td>
<td>$-2$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$-2$</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>$-1$</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

2 Write down the missing number.

a $2 \times (-3) = -6$, so $-6 \div (-3) = \boxed{2}$

b $2 \times (-3) = -6$, so $-6 \div 2 = \boxed{-3}$

c $-16 \div 4 = -4$, so $\boxed{-4} \times (-4) = 16$
3 Complete each sentence by inserting the missing word *positive or negative*.
   a The product \((\times)\) of two positive numbers is ________________.
   b The product \((\times)\) of two negative numbers is ________________.
   c The product \((\times)\) of two numbers with opposite signs is ________________.
   d The quotient \((\div)\) of two positive numbers is ________________.
   e The quotient \((\div)\) of two negative numbers is ________________.
   f The quotient \((\div)\) of two numbers with opposite signs is ________________.

4 Calculate the answer to these products.
<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3 \times (-5))</td>
<td>(1 \times (-10))</td>
<td>(-3 \times 2)</td>
<td>(-9 \times 6)</td>
</tr>
<tr>
<td>(-8 \times (-4))</td>
<td>(-2 \times (-14))</td>
<td>(-12 \times (-12))</td>
<td>(-11 \times 9)</td>
</tr>
<tr>
<td>(-13 \times 3)</td>
<td>(7 \times (-12))</td>
<td>(-19 \times (-2))</td>
<td>(-36 \times 3)</td>
</tr>
<tr>
<td>(-6 \times (-11))</td>
<td>(5 \times (-9))</td>
<td>(-21 \times (-3))</td>
<td>(-36 \times (-2))</td>
</tr>
</tbody>
</table>

5 Calculate the answer to these quotients.
<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(14 \div (-7))</td>
<td>(36 \div (-3))</td>
<td>(-40 \div 20)</td>
<td>(-100 \div 25)</td>
</tr>
<tr>
<td>(-9 \div (-3))</td>
<td>(-19 \div (-19))</td>
<td>(-25 \div 5)</td>
<td>(38 \div (-2))</td>
</tr>
<tr>
<td>(84 \div (-12))</td>
<td>(-108 \div 9)</td>
<td>(-136 \div 2)</td>
<td>(-1000 \div (-125))</td>
</tr>
<tr>
<td>(-132 \div (-11))</td>
<td>(-39 \div (-3))</td>
<td>(78 \div (-6))</td>
<td>(-156 \div (-12))</td>
</tr>
</tbody>
</table>

6 Work from left to right to find the answer. Check your answer using a calculator.
<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2 \times (-3) \times (-4))</td>
<td>(-1 \times 5 \times (-3))</td>
<td>(-10 \times 5 \times 2)</td>
<td>([-10 \times 5 \times 2])</td>
</tr>
<tr>
<td>(-15 \div (-3) \times 1)</td>
<td>(-2 \times 7 \div (-14))</td>
<td>(100 \div (-20) \times 2)</td>
<td>([-100 \div 25])</td>
</tr>
<tr>
<td>(48 \div (-2) \times (-3))</td>
<td>(-36 \times 2 \div (-4))</td>
<td>(-125 \div 25 \div (-5))</td>
<td>([-100 \div 25])</td>
</tr>
<tr>
<td>(-8 \div (-8) \div (-1))</td>
<td>(46 \div (-2) \times (-3) \times (-1))</td>
<td>(-108 \div (-12) \div (-3))</td>
<td>([-100 \div 25])</td>
</tr>
</tbody>
</table>

7 Write down the missing number in these calculations.
<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5 \times \square = -35)</td>
<td>(\square \times (-2) = -8)</td>
<td>(16 \div \square = -4)</td>
<td></td>
</tr>
<tr>
<td>(-32 \div \square = -4)</td>
<td>(\square \div (-3) = -9)</td>
<td>(\square \div 7 = -20)</td>
<td></td>
</tr>
<tr>
<td>(-5000 \times \square = -10000)</td>
<td>(-87 \times \square = 261)</td>
<td>(243 \div \square = -81)</td>
<td></td>
</tr>
<tr>
<td>(50 \div \square = 50)</td>
<td>(-92 \times \square = 184)</td>
<td>(-800 \div \square = -20)</td>
<td></td>
</tr>
</tbody>
</table>

8 Remember that \(\frac{9}{3}\) means \(9 \div 3\). Use this knowledge to simplify each of the following.
<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{12}{4})</td>
<td>(\frac{21}{-7})</td>
<td>(-\frac{40}{-5})</td>
<td>(-\frac{124}{-4})</td>
</tr>
<tr>
<td>(-\frac{15}{-5})</td>
<td>(-\frac{100}{-20})</td>
<td>(-\frac{900}{30})</td>
<td>(\frac{20000}{-200})</td>
</tr>
</tbody>
</table>

9 Given that \(3^2 = 3 \times 3 = 9\) and \((-3)^2 = -3 \times (-3) = 9\), simplify each of the following.
<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-2)^2)</td>
<td>((-1)^2)</td>
<td>((-9)^2)</td>
<td>((-10)^2)</td>
</tr>
<tr>
<td>((-6)^2)</td>
<td>((-8)^2)</td>
<td>((-3)^2)</td>
<td>((-1.5)^2)</td>
</tr>
</tbody>
</table>
10 List the different pairs of integers that multiply to give these numbers.
   a 6     b 16     c -5     d -24

11 Insert a multiplication or division sign between the numbers to make a true statement.
   a 2 □ -3 □ -6 = 1   b -25 □ -5 □ 3 = 15
   c -36 □ 2 □ -3 = 216  d -19 □ -19 □ 15 = 15

12 a There are two distinct pairs of numbers whose product is -8 and difference is 6. What are the two numbers?
   b The quotient of two numbers is -11 and their difference is 36. What are the two numbers?
      There are two distinct pairs to find.

13 Given that $2^4$ means $2 \times 2 \times 2 \times 2$ and $(-2)^4 = -2 \times -2 \times -2 \times -2$
   a Calculate:
      i $(-2)^3$     ii $(-2)^6$     iii $(-3)^3$     iv $(-3)^4$
   b Which questions from part a give positive answers and why?
   c Which questions from part a give negative answers and why?

14 $a \times b$ is equivalent to $ab$, and $2 \times (-3)$ is equivalent to $-(2 \times 3)$. Use this information to simplify these expressions.
   a $a \times (-b)$     b $-a \times b$     c $-a \times (-b)$

Enrichment: Multiplication and division with negative integers

15 $(-1) + (-2) + (-3) + (-4) = -10$ and $(-1) \times (-2) \times (-3) \times (-4) = 24$. Therefore, it is possible to use the numbers $-1, -2, -3$ and $-4$ to achieve a ‘result’ of $-10$ and $24$. What other ‘results’ can you find using those four numbers and any mathematical operations?

For example: What is $(-1) \times (-2) + (-3) \times (-4)$? Can you find expressions for every integer from $-20$ to $20$?
Order of operations with positive and negative integers

We have learnt from our study of positive whole numbers that there is a particular order to follow when dealing with mixed operations and brackets. This order also applies when dealing with negative numbers. For example: $-2 + 3 \times (-4)$ is different from $(-2 + 3) \times (-4)$.

Let’s start: Brackets or not?
During a classroom debate about the statement $3 \times (-4) - 8 \div (-2) = -8$:
- Lil says that the statement needs to have brackets to make it true.
- Max says that even with brackets it is impossible to make it true.
- Riley says that it is correct as it is and there is no need for brackets.
Who is correct and why?

Key ideas
- When working with more than one operation and with positive and/or negative numbers:
  - Deal with brackets first.
  - Do multiplication and division next, working from left to right.
  - Do addition and subtraction last, working from left to right.

Example 7 Using order of operations
Use order of operations to evaluate the following.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong></td>
<td>$5 + 2 \times (-3)$</td>
</tr>
<tr>
<td><strong>b</strong></td>
<td>$-6 \times 2 - 10 \div (-5)$</td>
</tr>
</tbody>
</table>

**SOLUTION**

**a**  
$5 + 2 \times (-3) = 5 + (-6)$  
$= -1$

**b**  
$-6 \times 2 - 10 \div (-5) = -12 - (-2)$  
$= -12 + 2$  
$= -10$

**EXPLANATION**

**a**  
Do the multiplication before the addition.

**b**  
Do the multiplication and division first.  
When subtracting $-2$, add its opposite.
Example 8 Order of operations with brackets

Use order of operations to evaluate the following.

\[
\begin{align*}
a &\quad (-2 - 1) \times 8 \\
&\quad = -3 \times 8 \\
&\quad = -24 \\
b &\quad 5 \div (-10 + 5) + 5 \\
&\quad = 5 \div (-5) + 5 \\
&\quad = -1 + 5 \\
&\quad = 4 \\
c &\quad -6 - \frac{10 + 2}{3} \\
&\quad = -6 - \frac{12}{3} \\
&\quad = -6 - 4 \\
&\quad = -10
\end{align*}
\]

EXPLANATION

a Deal with brackets first.

b Deal with brackets first. Then do the division before the subtraction.

c Simplify the fraction first.

Exercise 3E

1 Which operation (i.e. addition, subtraction, multiplication or division) is done first in each of the following problems?

\[
\begin{align*}
a &\quad -2 + 2 + 1 \\
b &\quad 8 \times (-6) - 4 \\
c &\quad -3 + 2 \times (-6) \\
d &\quad 7 - (-8) + 4 \\
e &\quad (-2 + 3) + 5 \\
f &\quad -6 \div (4 - (-2)) \\
g &\quad -4 \times 3 + (-6) \\
h &\quad (2 + 3 \times (-2)) + 1 \\
i &\quad -11 + (7 - 2 \times (-2))
\end{align*}
\]

2 Classify each of the following statements as true or false.

\[
\begin{align*}
a &\quad -4 + 2 \times 3 = -4 + (2 \times 3) \\
b &\quad -4 + 2 \times 3 = (-4 + 2) \times 3 \\
c &\quad 8 \times (2 - (-2)) = 8 \times 4 \\
d &\quad 8 \times (2 - (-2)) = 8 \times 0 \\
e &\quad -40 - 20 \div (-5) = (-40 - 20) \div (-5) \\
f &\quad -40 - 20 \div (-5) = -40 - (20 \div (-5))
\end{align*}
\]

3 Use order of operations to evaluate the following. Check your answer using a calculator.

\[
\begin{align*}
a &\quad 2 + 3 \times (-3) \\
b &\quad 9 + 10 + (-5) \\
c &\quad 20 + (-4) \times 4 \\
d &\quad 18 + (-9) \times 1 \\
e &\quad 10 - 2 \times (-3) \\
f &\quad 10 - 1 \times (-4) \\
g &\quad -8 - (-7) \times 2 \\
h &\quad -2 \times 4 + 8 \times (-3) \\
i &\quad -3 \times (-1) + 4 \times (-2) \\
j &\quad 12 + (-6) + 4 \div (-2) \\
k &\quad -30 + 5 - 6 \times 2 \\
l &\quad -2 \times 3 - 4 \div (-2) \\
m &\quad 8 \times (-2) - (-3) \times 2 \\
n &\quad -1 \times 0 - (-4) \times 1 \\
o &\quad 0 \times (-3) - (-4) \times 0 + 0
\end{align*}
\]
4. Use order of operations to evaluate the following. Check your answer using a calculator.

   a. \((3 + 2) \times (-2)\)
   b. \(\frac{8 - 4}{-2}\)
   c. \(-3 \times (-2 + 4)\)
   d. \(-1 \times (7 - 8)\)
   e. \(\frac{10}{4 - (-1)}\)
   f. \((2 + (-3)) \times (-9)\)
   g. \(\frac{24 - 12}{16 + (-4)}\)
   h. \((3 - 7) + (-1 + 0)\)
   i. \(-2 \times (8 - 4) + (-6)\)
   j. \(-2 - 3 \times (-1 + 7)\)
   k. \(0 + (-2) + (1 - 2)\)
   l. \(1 - \frac{2 \times (-3)}{-3 - (-2)}\)
   m. \((-3 + (-5)) \times (-2 - (-1))\)
   n. \(\frac{-3}{-1 + 4} \times 6\)
   o. \(-5 - (8 + (-2)) + 9 \div (-9)\)

5. A shop owner had bought socks at $5 a pair but, during an economic downturn, sold them for $3 a pair. In a particular week, 124 pairs are sold and there are other costs of $280. What is the shop owner’s overall loss for the week?

6. A debt of $550 is doubled and then $350 of debt is removed each month for 3 months. What is the final balance?

7. Insert brackets to make each statement true.

   a. \(-2 + 3 \times 8 = 8\)
   b. \(-10 + 4 + 1 = -2\)
   c. \(-1 + 7 \times 2 - 15 = -3\)
   d. \(-5 - 1 \div (-6) = 1\)
   e. \(3 - 8 \div 5 + 1 = 0\)
   f. \(50 \times 7 - 8 \times (-1) = 50\)
   g. \(-2 \times 3 - (-7) - 1 = -21\)
   h. \(-3 + 9 \div (-7) + 5 = -3\)
   i. \(32 - (-8) + (-3) + 7 = 10\)

8. By inserting only one pair of brackets, how many different answers are possible for this calculation? Also include the answers for which brackets are not used.
   
   \(-2 + 8 \times (-4) - (-3)\)
9 If brackets are removed from these problems, does the answer change?

a \((2 \times 3) - (-4)\)  
b \((8 + (-2)) - 1\)  
c \((-2 + 3) \times 4\)  
d \(9 \div (-4 + 1)\)  
e \((9 - (-3) \times 2) + 1\)  
f \((-1 + 8 + (-2)) \times 2\)

10 State if each of the following is generally true or false.

a \((-3 + 1) + (-7) = -3 + (1 + (-7))\)  
b \((-3 + 1) - (-7) = -3 + (1 - (-7))\)  
c \((a + b) + c = a + (b + c)\)  
d \((a + b) - c = a + (b - c)\)  
e \((a - b) + c = a - (b + c)\)  
f \((a - b) - c = a - (b - c)\)

11 a Given that \(5^3 = 5 \times 5 \times 5\), is the answer to each of the following positive or negative?

i \(-6 \times (-4) \times (-8) \times (-108) \times (-96)\)  
ii \(-100 \div (-2) + 2 + (-5)\)  
iii \((-3)^3\)  
iv \(-1 \times (-2)^3\)  
v \(\frac{-6 \times (-3) \times 4 \times 7 \times (-3)}{(-2)^2}\)  
vi \(\frac{(-1)^2 \times (-1)}{(-1)^3 \times (-1)}\)

b Explain the strategy you used to answer the questions in part a.

Enrichment: Powers and negative numbers

12 First, note that:

- \(2^4 = 2 \times 2 \times 2 \times 2 = 16\)
- \((-2)^4 = -2 \times (-2) \times (-2) \times (-2) = 16\)
- \(-2^4 = -(2 \times 2 \times 2 \times 2) = -16\)

When evaluating expressions with powers, the power is dealt with first in the order of operations.

For example: \((-2)^3 - 1) \div (-3) = (-8 - 1) \div (-3) = -9 \div (-3) = 3\)

Evaluate each of the following.

a \(2^2\)  
b \((-2)^3\)  
c \(-2^2\)  
d \((-2)^5\)  
e \(-2^5\)  
f \((3^2 - 1) \times 4\)  
g \((-3)^3 - 1) \div (-14)\)  
h \(30 \div (1 - 4^2)\)  
i \(-10\,000 \div (-10)^4\)

13 Kevin wants to raise \(-3\) to the power of 4. He types \(-3^4\) into a calculator and gets \(-81\). Explain what Kevin has done wrong.
The Cartesian plane

During the 17th century, two well-known mathematicians, René Descartes and Pierre de Fermat, independently developed the idea of a number plane. The precise positions of points are illustrated using coordinates, and these points can be plotted using the axes as measuring guides. This invention revolutionised the study of mathematics and provided a vital link between geometry and algebra. The number plane, or coordinate plane, is also called the Cartesian plane (named after Descartes). It uses two axes at right angles that extend in both the positive and negative directions.

Let’s start: North, south, east and west

The units for this grid are in metres.

René starts at position O and moves:
- 3 m east
- 2 m south
- 4 m west
- 5 m north.

Pierre starts at position O and moves:
- 1 m west
- 3 m south
- 4 m east
- 5 m north.

Using the number plane, how would you describe René and Pierre’s final positions?

Key ideas

- The Cartesian plane or number plane uses two axes (x-axis and y-axis) at right angles. Each axis uses a scale that includes both positive and negative numbers.

- A point plotted on the plane has an x and y coordinate, which is written as (x, y). The x coordinate is written before the y coordinate, as in the alphabet.

- The point (0, 0) is called the origin or O.
Example 9 Finding coordinates

For the Cartesian plane shown, write down the coordinates of the points labelled $A$, $B$, $C$ and $D$.

\[ A = (1, 1) \]
\[ B = (3, -2) \]
\[ C = (-2, -4) \]
\[ D = (-3, 3) \]

For each point, write the $x$ coordinate first (from the horizontal axis) followed by the $y$ coordinate (from the vertical axis).

Exercise 3F

1. Match the points $A$, $B$, $C$, $D$, $E$, $F$, $G$ and $H$ with the given coordinates.

   a. $(-1, 3)$  
   b. $(2, -3)$  
   c. $(2, 1)$  
   d. $(-2, -2)$  
   e. $(3, 3)$  
   f. $(-3, 1)$  
   g. $(1, -2)$  
   h. $(-1, -1)$

2. Count the number of points, shown as dots, on this plane that have:
   
   a. both $x$ and $y$ coordinates as positive numbers
   b. an $x$ coordinate as a positive number
   c. a $y$ coordinate as a positive number
   d. an $x$ coordinate as a negative number
   e. a $y$ coordinate as a negative number
   f. both $x$ and $y$ coordinates as negative numbers
   g. neither $x$ nor $y$ as positive or negative numbers
Example 3

For the Cartesian plane given, write down the coordinates of the points labelled A, B, C, D, E, F, G and H.

4 a Draw a set of axes, using 1 cm spacings. Use −4 to 4 on both axes.
b Now plot these points.
   i (−3, 2)
   ii (1, 4)
   iii (2, −1)
   iv (−2, −4)
   v (2, 2)
   vi (−1, 4)
   vii (−3, −1)
   viii (1, −2)

5 For the number plane given, write down the coordinates of the points labelled A, B, C, D, E, F, G and H.

6 Seven points have the following x and y coordinates.

<table>
<thead>
<tr>
<th>x</th>
<th>−3</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>−2</td>
<td>−1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

   a Plot the seven points on a Cartesian plane. Use −3 to 3 on the x-axis and −2 to 4 on the y-axis.
b What do you notice about these seven points on the Cartesian plane?

7 Seven points have the following x and y coordinates.

<table>
<thead>
<tr>
<th>x</th>
<th>−3</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>−1</td>
<td>−3</td>
<td>−5</td>
<td>−7</td>
</tr>
</tbody>
</table>

   a Plot the seven points on a number plane. Use −3 to 3 on the x-axis and −7 to 5 on the y-axis.
b What do you notice about these seven points on the number plane?
8 When plotted on the Cartesian plane, what shape does each set of points form?
   a  \(A(-2, 0), B(0, 3), C(2, 0)\)
   b  \(A(-3, -1), B(-3, 2), C(1, 2), D(1, -1)\)
   c  \(A(-4, -2), B(3, -2), C(1, 2), D(-1, 2)\)
   d  \(A(-3, 1), B(-1, 3), C(4, 1), D(-1, -1)\)

9 Using the origin as one corner, the point \(A(3, 2)\) as the opposite corner and the axes as two of the sides, a rectangle can be positioned on a set of axes, as shown opposite. Its area is 6 square units. Find the area of the rectangle if the point \(A\) is:
   a  \((2, 2)\)
   b  \((-3, 2)\)
   c  \((-1, -4)\)
   d  \((3, -5)\)

10 Karen’s bushwalk starts at a point \((2, 2)\) on a grid map. Each square on the map represents 1 kilometre. If Karen walks to the points \((2, -7)\), then \((-4, -7)\), then \((-4, 0)\) and then \((2, 0)\), how far has she walked in total?

11 The points \(A(-2, 0), B(-1, ?)\) and \(C(0, 4)\) all lie on a straight line. Find the \(y\) coordinate of point \(B\).

12 The points \(A(-4, 8), B(-1, ?)\) and \(C(2, -2)\) all lie on a straight line. Find the \(y\) coordinate of point \(B\).

13 Consider the points \(A(-2, 2), B(0, 2)\) and \(C(3, -2)\).
   a  Which point is closest to \((0, 0)\)?
   b  Which point is farthest from \((0, 0)\)?
   c  List the given points in order from closest to farthest from the origin, \(O\).

14 A point \((a, b)\) sits on the number plane in one of the four regions 1, 2, 3 or 4, as shown. These regions are called quadrants.
   a  Name the quadrant or quadrants that include the points where:
      i  \(a > 0\)
      ii  \(a > 0\) and \(b < 0\)
      iii  \(b < 0\)
      iv  \(a < 0\) and \(b < 0\)
   b  Shade the region that includes all points for which \(b > a\).

15 Consider the points \(A(0, 0)\) and \(B(3, 1)\).
   a  \(ABCD\) is a square. Write down the coordinates of \(C\) and \(D\) if \(C\) is in the first quadrant.
   b  \(ABE\) is an isosceles right-angled triangle. There are four possible locations for point \(E\) if \(AB\) is not the hypotenuse. List them all.
   c  \(G\) is the point \((1, 3)\) and \(ABGH\) is a parallelogram. Write down the coordinates of \(H\).
Account balance with spreadsheets

If you have money saved in a bank account, your account balance should be positive. If you take out or spend too much money, your account balance may become negative.

a Set up a spreadsheet to record and calculate a bank balance. Enter the given information describing one week of deposits and withdrawals, as shown.

b i For the given spreadsheet, what is the balance at the end of May 1st?
   ii On which day does the balance become negative?

c Enter this formula into cell E5:  = E4+C5–D5
   Fill down to reveal the balance after each day.

d Enter another week of deposits and withdrawals so that the balance shows both positive and negative amounts.

e Now alter your opening balance. What opening balance is needed so that the balance never becomes negative? Is there more than one value? What is the least amount?

f Investigate how positive and negative numbers are used on credit card accounts. Give a brief explanation.
1 Complete these magic squares. All rows, columns and diagonals sum to the same number.

\[
\begin{array}{ccc}
-5 & & \\
0 & & \\
-6 & -1 & \\
\end{array}
\quad \begin{array}{ccc}
-9 & 5 & -6 \\
-4 & -1 & \\
1 & & \\
3 & -8 & 6 \\
\end{array}
\quad \begin{array}{ccc}
 & & -1 \\
 & -3 & \\
-7 & -5 & -4 \\
-2 & -13 & 1 \\
\end{array}
\]

2 Find the next three numbers in these patterns.

\[
a 3, -9, 27, \ldots, \\
b -32, 16, -8, \ldots, \\
c 0, -1, -3, -6, \ldots, \\
d -1, -1, -2, -3, -5, \ldots,
\]

3 Evaluate the following.

\[
a -100 + (-98) + (-96) + \ldots + 98 + 100 \\
b (50 - 53) + (49 - 52) + (48 - 51) + \ldots + (0 - 3)
\]

4 Insert brackets and symbols (+, −, ×, ÷) into these number sentences to make them true.

\[
a -3 \quad 4 \quad -2 = -6 \\
b -2 \quad 5 \quad -1 \quad 11 = 21 \\
c 1 \quad 30 \quad -6 \quad -2 = -3
\]

5 a The difference between two numbers is 14 and their sum is 8. What are the two numbers?

b The difference between two numbers is 31 and their sum is 11. What are the two numbers?

6 Place the integers −3, −2, −1, 0, 1 and 2 into the triangle so that the sum of every side is:

\[
\begin{array}{ccc}
& & \\
& & \\
-3 & & \\
0 & & \\
& -2 & \\
\end{array}
\]
Chapter summary

Integers
..., −3, −2, −1, 0, 1, 2, 3, ...

Adding and subtracting positive integers
−3 + 5 = 2
−4 + 3 = −1
5 − 7 = −2
−1 − 10 = −11

Adding and subtracting negative integers
2 + (−3) = 2 − 3 = −1
−5 + (−4) = −5 − 4 = −9
4 − (−3) = 4 + 3 = 7
−10 − (−6) = −10 + 6 = −4

Multiplication
2 × 3 = 6
2 × (−3) = −6
−2 × 3 = −6
−2 × (−3) = 6

Division
10 ÷ 5 = 2
10 ÷ (−5) = −2
−10 ÷ 5 = −2
−10 ÷ (−5) = 2

Order of operations
First brackets, then × or ÷ then + or −, from left to right.
3 × (5 − (−2)) + 8 ÷ (−4) = 3 × 7 + (−2) = 21 + (−2) = 19

Number line
negative
−3 −2 −1 0 1 2 3
−2 < 3 1 > −1

Cartesian plane
y
3
2
1
0
x
−3 −2 −1 0 1 2 3
(−3, 0) (3, 1)
(−2, −2) (1, −2)
(0, 2)
Multiple-choice questions

1 When the numbers −4, 0, −1, 7 and −6 are arranged from lowest to highest, the correct sequence is:
   A 0, −1, −4, −6, 7  B 0, −4, −6, −1, 7  C −6, −4, −1, 0, 7  D −1, −4, −6, 0, 7  E −6, −1, 0, −4, 7

2 The difference between −19 and 8 is:
   A 152  B −11  C −27  D 11  E 27

3 The missing number in 2 − □ = 3 is:
   A 1  B −1  C 5  D −5  E 2

4 5 − (−2) + (−7) is equal to:
   A −4  B 10  C 7  D 0  E 14

5 The temperature inside a mountain cabin is initially −5°C. After burning a fire for 2 hours the temperature rises to 17°C. What is the rise in temperature?
   A −12°C  B 12°C  C 22°C  D −85°C  E −22°C

6 The product and quotient of two negative numbers is:
   A positive  B negative  C zero  D added  E different

7 −2 × (−5) ÷ (−10) is equal to:
   A −5  B 10  C −20  D 1  E −1

8 Which operation (i.e. addition, subtraction, multiplication or division) is completed second in the calculation of (−2 + 5) × (−2) + 1?
   A addition  B subtraction  C multiplication  D division  E brackets

9 (−2) × 5 − (−2) is equal to:
   A −12  B −8  C 8  D 12  E 9

10 The points A(−2, 3), B(−3, −1), C(1, −1) and D(0, 3) are joined on a number plane. What shape do they make?
    A triangle  B square  C trapezium  D kite  E parallelogram
Chapter 3 Computation with positive and negative integers

Short-answer questions

1 Insert the symbol < (is less than) or > (is greater than) into each statement to make it true.

   a 0 < 7   b −1 > 4   c 3 < −7   d −11 < −6

2 Evaluate:

   a 2 − 7   b −4 + 2   c 0 − 15   d −36 + 37
   e 5 + (−7)   f −1 + (−4)   g 10 − (−2)   h −21 − (−3)
   i 1 − 5 + (−2)   j −3 + 7 − (−1)   k 0 + (−1) − 10   l −2 − (−3) − (−4)

3 Find the missing number for each of the following.

   a −2 + □ = −3   b −1 + □ = −10   c 5 − □ = 6   d −2 − □ = −4
   e −1 − □ = 20   f −15 − □ = −13   g 7 + □ = −80   h −15 + □ = 15

4 Evaluate:

   a 5 × (−2)   b −3 × 7   c −2 × (−15)   d 10 ÷ (−2)
   e −36 ÷ 12   f −100 ÷ (−25)   g −3 × 2 ÷ (−6)   h −38 ÷ (−19) × (−4)

5 Find the missing number.

   a 4 × □ = −8   b □ ÷ −5 = 10   c □ ÷ 9 = −4   d −1 × □ = 1

6 Use order of operations to find the answers to these expressions.

   a −2 + 5 × (−7)   b −1 − 18 ÷ (−2)   c −15 ÷ (1 + 4)
   d 5 − 4 × (−3) ÷ (−3)   e (−2 − 5) × (8 ÷ (−1))   f −7 × ((−4) − 7) + 3

7 Evaluate:

   a −3 − 3 − 3   b (1 − 2) × 3 − 4

8 For the Cartesian plane shown, write down the coordinates of the points labelled A, B, C, D, E and F.
Extended-response questions

1 A scientist, who is camped on the ice in Greenland, records the following details in her notepad regarding the temperature over five days. Note that ‘min’ stands for minimum and ‘max’ stands for maximum.

• Monday: min = −18°C, max = −2°C.
• Decreased 29°C from Monday’s max to give Tuesday’s min.
• Wednesday’s min was −23°C.
• Max was only −8°C on Thursday.
• Friday’s min is 19°C colder than Thursday’s max.

a What is the overall temperature increase on Monday?
b What is Tuesday’s minimum temperature?
c What is the difference between the minimum temperatures for Tuesday and Wednesday?
d What is the overall temperature drop from Thursday’s maximum to Friday’s minimum?
e By how much will the temperature need to rise on Friday if its maximum is 0°C?

2 When joined, these points form a picture on the number plane. What is the picture?
A(0, 5), B(1, 3), C(1, 1), D(2, 0), E(1, 0), F(1, −2), G(3, −5), 
H(−3, −5), I(−1, −2), J(−1, 0), K(−2, 0), L(−1, 1), M(−1, 3), N(0, 5)